

Dynamic generation of orbital quasiparticle entanglement in mesoscopic conductors

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We propose a scheme for dynamically creating orbitally entangled electron-hole pairs through a time-dependent variation of the electrical potential in a mesoscopic conductor. The time-dependent potential generates a superposition of electron-hole pairs in two different orbital regions of the conductor, a Mach-Zehnder interferometer in the quantum Hall regime. The orbital entanglement is detected via violation of a Bell inequality, formulated in terms of zero-frequency current noise. Adiabatic cycling of the potential, both in the weak and strong amplitude limit, is considered.

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Entanglement is an aspect of quantum mechanics which has intrigued and fascinated physicists for seven decades.¹ While the initial interest mostly concerned the interpretation and fundamental properties of quantum mechanics,^{2,3} recently entanglement has emerged as a resource for quantum information and computation purposes. A controlled creation, manipulation and detection of quasiparticle entanglement in mesoscopic conductors is therefore of interest. In particular, computation is a time-dependent process with elementary steps often driven by a masterclock.⁴ Clearly it would be desirable to be able to produce entanglement on command, once per clock-cycle.

Here we propose, as a first step towards time-controlled quasiparticle entanglement, a scheme for the dynamic generation of orbitally entangled⁵ quasiparticle pairs in a mesoscopic conductor. Orbital entanglement^{5,6,7,8,9} uses quasiparticles generated by two spatially separated sources. The spatial index of the sources plays the role of a pseudo-spin index. Due to indistinguishability of the pair creation events, the two sources generate entangled electron-electron⁵ or electron-hole⁸ states. The main advantages of orbital entanglement are that local rotations of the orbital single particle state can be performed with experimentally available^{10,11} electronic beamsplitters and that detection, in contrast to spin entanglement, can be done without spin-to-charge conversion.

In the set-up considered here the sources are provided by electrical potentials that are varied periodically and adiabatically in two spatially separated regions in the conductor, a Mach-Zehnder interferometer in the quantum Hall regime (see Fig. 1). The oscillating potentials excite electron-hole pairs, orbitally entangled with respect to the regions of emission. The emitted quasiparticles are detected in electronic reservoirs, all kept at zero bias. Our scheme bears some resemblance to quantum pumping^{12,13,14,15} but due to the chiral nature of the geometry no net electrical current per clock-cycle is generated. Instead the excited quasiparticles give rise to current noise. Shot noise in the absence of dc-current has recently been measured in an experiment which excites quasiparticles with the help of an oscillating contact potential.¹⁶ We show that the entanglement can be detected via a Bell Inequality formulated in terms

of the period-averaged, zero frequency noise. Although the scheme is non-ideal, i.e. it generates less than one entangled pair per cycle, it is a simple and transparent proposal of entanglement generated by time-dependent electrical potentials. Moreover, the geometry is similar to a Mach-Zehnder interferometer recently realized experimentally,¹⁷ making our proposal experimentally highly relevant.

There are to date a large number of suggestions for creation of spin^{18,19} as well as orbital⁵ entanglement in quantum dot,¹⁸ superconductor,^{5,19,20} quantum Hall^{6,8,9} and beam-splitter^{21,22} systems. A common feature of the majority of the proposed schemes is that the entangled quasiparticles are emitted in a random and uncontrolled fashion, by e.g. tunneling across a potential barrier. This is similar to the creation of entangled photon-pairs by

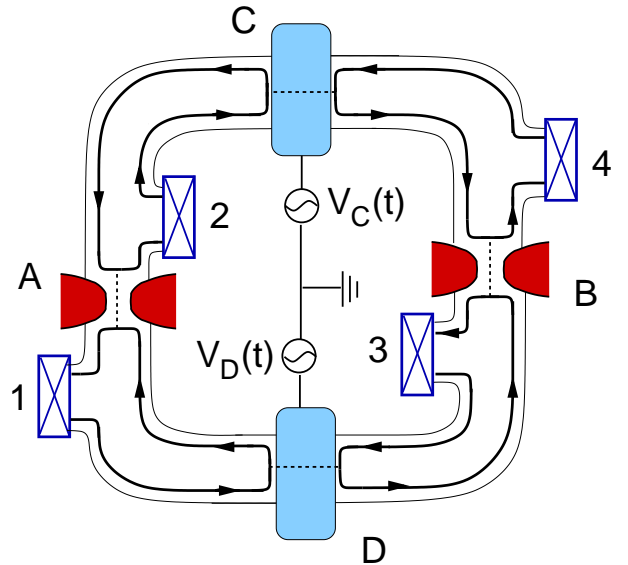


FIG. 1: Mach-Zehnder geometry in the quantum Hall regime. Transport takes place along a single edge state (thick black line) in the direction shown by the arrows. The potentials $V_C(t)$ and $V_D(t)$ at C and D are adiabatically and periodically modulated in time, creating electron-hole pairs propagating towards reservoirs 1 to 4. The static point contacts at A and B work as controllable beam-splitters.

parametric down-conversion²³ in optics. Although this form of entanglement is useful for basic quantum information processes, for more complex tasks such as quantum computation it is however desirable to have a time-controlled source of entangled quasiparticles.

In our proposal for dynamic generation of entanglement (see Fig. 1) we consider a Mach-Zehnder interferometer in the quantum Hall regime, connected to four reservoirs, 1 to 4, in thermal equilibrium. Transport takes place along a single, spin-polarized edge state. Due to the chiral nature of the edge state transport,²⁴ backscattering²⁵ takes place only at the scatterers A, B, C and D . At A and B quantum point contacts work as controllable beam-splitters.¹⁰ At C and D (electrostatic top- or side gates) the corresponding electrical potentials $V_C(t)$ and $V_D(t)$ are modulated periodically in time, with a period $\tau = 2\pi/\omega$. The adiabatic limit is considered, where the amplitudes for scattering between the reservoirs 1 to 4 are independent on energy on the scale of the clock frequency ω . Only results to first order in ω are discussed and the temperature is taken much smaller than $\hbar\omega$.

We point out that quasiparticle entanglement in static, voltage-biased quantum Hall systems have been investigated previously in Refs. [6,7,8,9]. Here we instead consider a situation where all reservoirs are kept at zero bias but the scattering potentials for the electrons in regions C and D are time-dependent. To consider the simplest possible system, quasiparticles are, in contrast to Refs. [6,7,8,9], injected and detected at the same reservoirs. This however makes the scheme in Fig. 1 less nonlocal.

Due to the oscillating potentials at C and D , electrons incident from the reservoirs 1 to 4 can absorb or emit one or several quanta of energy $\hbar\omega$ before propagating out to the reservoirs again. In this Floquet picture^{13,14} the scattering in both energy and real space can be described by scattering matrices

$$S_{C/D}(E_n, E) = \begin{pmatrix} r_{C/D}(E_n, E) & t'_{C/D}(E_n, E) \\ t_{C/D}(E_n, E) & r'_{C/D}(E_n, E) \end{pmatrix} \quad (1)$$

where e.g. $t_C(E_n, E)$ is the amplitude for an electron incoming at energy E from left towards C to be transmitted to the right at energy $E_n = E + n\hbar\omega$. The dependence of the scattering amplitudes on $V_{C/D}(t)$ is determined by the properties of the scattering potential in the regions C and D , here we only work with the scattering amplitudes themselves to keep maximum generality.

We first focus on the limit of weakly oscillating potentials, $V_{C/D}(t) = V_{C/D} + \delta V_{C/D} \cos(\omega t + \phi_{C/D})$, with $\delta V_{C/D}$ so small that only the amplitudes to absorb or emit one quanta need to be taken into account. The relevant scattering amplitudes are then e.g. $t_C \equiv t_C(E, E)$ and $\delta t_C^\pm = \delta t_C \exp(\mp i\phi_C) \equiv t_C(E_{\pm 1}, E)$, with $\delta t_C = \delta V_C(\partial t_C / \partial V_C)/2$, and similar for the other amplitudes. Formally the weak potential limit, $|\delta t_C|^2 \ll 1$ etc, implies that the scattering amplitudes are weakly dependent on energy on the scale of the potential variations $\delta V_{C/D}$.

To demonstrate that entanglement is created by the oscillating potentials, we first consider the state of the particles emitted from the two regions C and D . It can be constructed from the many-body state of the electrons incident from the reservoirs 1 to 4, (suppressing spin) $|\Psi_{in}\rangle = \prod_{j=1}^4 \prod_E a_j^\dagger(E) |0\rangle$, where $|0\rangle$ is the true vacuum and a_j^\dagger creates an electron at energy E , incident from reservoir j . Introducing operators, e.g. $b_{AC}^\dagger(E)$ creating an outgoing electron at energy E at contact C propagating towards contact A , we can relate the b -operators to the a -operators at C as

$$\begin{pmatrix} b_{AC}(E) \\ b_{BC}(E) \end{pmatrix} = \sum_{n=0, \pm 1} S_C(E, E_n) \begin{pmatrix} a_2(E_n) \\ a_4(E_n) \end{pmatrix} \quad (2)$$

and similarly at D . Inserting these relations into $|\Psi_{in}\rangle$ and expanding to first order $\delta V_{C/D}$, we find the state outgoing from C and D in terms of the b -operators as

$$|\Psi_{out}\rangle = |\bar{0}\rangle + \int_{-\hbar\omega}^0 dE (|\Psi_{out}^C(E)\rangle + |\Psi_{out}^D(E)\rangle) \quad (3)$$

with

$$|\Psi_{out}^C(E)\rangle = \sum_{\alpha, \beta=A, B} f_{\alpha\beta}^C b_{\alpha C}^\dagger(E_1) b_{\beta C}(E) |\bar{0}\rangle \quad (4)$$

where $f_{AA}^C = \delta r_C^+ r_C^* + \delta t_C^{++} t_C^*$, $f_{BB}^C = \delta t_C^{++} t_C^* + \delta r_C^{++} r_C^*$, $f_{AB}^C = \delta r_C^+ t_C^* + \delta t_C^{++} r_C^*$, $f_{BA}^C = \delta t_C^+ r_C^* + \delta r_C^{++} t_C^* = -e^{-2i\phi_C} (f_{AB}^C)^*$ and $|\Psi_{out}^D\rangle = |\Psi_{out}^C\rangle$ with $C \rightarrow D$. The ground state in terms of outgoing operators is $|\bar{0}\rangle = \prod_E b_{AC}^\dagger(E) b_{BC}^\dagger(E) b_{AD}^\dagger(E) b_{BD}^\dagger(E) |0\rangle$. Each term in Eq. (4) contains an operator product $b_{\alpha C}^\dagger(E_1) b_{\beta C}(E)$ acting on $|\bar{0}\rangle$, describing the destruction of one electron at an energy $-\hbar\omega < E < 0$ below the Fermi surface, i.e. the creation of a hole, and the creation of an electron at energy $0 < E_1 < \hbar\omega$ above (in leads αC and βC respectively). The effect of the weak potential oscillations is thus to create electron-hole pair excitations out of the

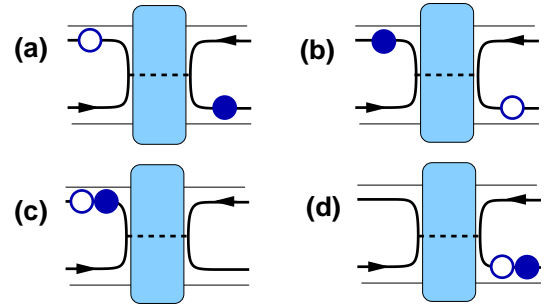


FIG. 2: The four different electron-hole pair emission processes at C : In the two upper processes, the pair is split and (a) the electron is emitted towards B and the hole towards A , or (b) the electron towards A and the hole towards B . In the two lower processes, both quasiparticles are emitted towards (c) A or (d) towards B . The same processes occur at D .

ground state.²⁶ The four different scattering processes at each contact C and D are shown schematically in Fig. 2.

The excited state in Eq. (3) is clearly orbitally entangled, it is a linear superposition of electron-hole pair wavepackets emitted at C and D , i.e. the contact indexes C and D form an orbital two-level system, or qubit. In first quantization, for identical scattering amplitudes at C and D , the state can be written $(|CC\rangle + |DD\rangle) \otimes |\bar{\Psi}\rangle$, where $|CC\rangle + |DD\rangle$ is maximally entangled, an orbital Bell state, and $|\bar{\Psi}\rangle$ contains all additional properties of the state, e.g. energy dependence and quasi-particle character. We note that the emitted wave-packets contain quasiparticles in the energy range $\hbar\omega$. The clock frequency thus, in this respect, plays the same role as the applied voltage in the entanglement schemes in e.g. Refs [5,6,8,9,21]. However, due to the absence of an applied bias there is no need to reformulate the ground state to reveal the entanglement, as was done in Ref. [5,6,8,9].

The entanglement is detected via cross-correlations of electrical currents flowing in leads 1, 2 (at A) and 3, 4 (at B). A number of works have investigated the noise properties of pumped mesoscopic conductors, see e.g. Refs. [14,15]. The operators b_j for outgoing electrons at reservoirs $j = 1$ to 4 are related to the $b_{\alpha\beta}$ operators via the (energy-independent) scattering matrices of the static, controllable point contact at A as

$$\begin{pmatrix} b_1(E) \\ b_2(E) \end{pmatrix} = \begin{pmatrix} \cos\theta_A & \sin\theta_A \\ -\sin\theta_A & \cos\theta_A \end{pmatrix} \begin{pmatrix} b_{AC}(E) \\ b_{AD}(E) \end{pmatrix} \quad (5)$$

and similarly at B . The current operator is $I_j(t) = (e/h) \int dE dE' \exp(i[E - E']t/\hbar) [b_j^\dagger(E) b_j(E') - a_j^\dagger(E) a_j(E')]$. The average current $\langle I_j(t) \rangle$ contains in general a dc- and an ac-part. However, the dc-current in the geometry under consideration is zero, i.e. there is no dc-electrical current, only quasiparticle current is generated. This can be understood qualitatively from the fact that the scattering amplitudes of the injected electrons, as a consequence of the chiral transport, only depend on one scattering potential, at C or D . Since adiabatic one-parameter pumps^{12,13} do not pump any net current, the dc current in our geometry is zero. The ac-current at contact j is proportional to the amplitude for both quasiparticles in an electron-hole pair emitted at C or D to be scattered to reservoir j . This provides however no information about the entanglement.

The cross-correlation between the currents $i = 1, 2$ and $j = 3, 4$, averaged over the time difference t' , is given by

$$S_{ij}(t) = \int dt' \langle \Delta I_i(t) \Delta I_j(t+t') + \Delta I_j(t+t') \Delta I_i(t) \rangle \quad (6)$$

with $\Delta I_j(t) = I_j(t) - \langle I_j(t) \rangle$. The noise, just as the current, has a dc and an ac-part, i.e. $S_{ij}(t) = S_{ij}^{dc} + S_{ij}^{ac}(t)$. The dc-part is, for e.g. reservoirs 1 and 3

$$S_{13}^{dc} = \frac{e^2}{\tau} [|f_{AB}^C \sin\theta_A \sin\theta_B + f_{AB}^D \cos\theta_A \cos\theta_B|^2 + |f_{BA}^C \sin\theta_A \sin\theta_B + f_{BA}^D \cos\theta_A \cos\theta_B|^2] \quad (7)$$

This expression has a simple physical explanation. The first term in the bracket in Eq. (7) is the amplitude for an emitted electron-hole pair from C or D to split, with the electron ending up in reservoir 1 and the hole in 3. The second term is just the amplitude for the opposite process, the electron detected in 3 and the hole in 1. The other correlators S_{ij}^{dc} are found similarly. We note that only the scattering processes where the pair splits [(a) and (b) in Fig. 2] contribute to the leading order cross-correlators.⁵

Considering for simplicity the case where the two scattering potentials at C and D are equal, i.e. $t_C = t_D = t$ etc, up to the pump phases $\phi_{C/D}$, one can write

$$S_{13}^{dc} = S_0 [\cos^2\theta_A \cos^2\theta_B + \sin^2\theta_A \sin^2\theta_B + 2\gamma \cos\theta_A \cos\theta_B \sin\theta_A \sin\theta_B] \quad (8)$$

with $S_0 = (2e^2/\tau) |\delta r t^* + \delta t' r'^*|^2$, $\gamma = \cos\varphi \cos(\phi_C - \phi_D)$ and $S_{24}^{dc} = S_{13}^{dc}$ and $S_{14}^{dc} = S_{23}^{dc} = S_{13}^{dc}(\theta_B \rightarrow \theta_B + \pi/2)$. Here φ is an overall phase containing possible scattering phases of contacts A and B and phases due to propagation along the edge states, including Aharonov-Bohm phases. The noise correlator is proportional to $|\delta r t^* + \delta t' r'^*|^2$, i.e. proportional to δV^2 . The last term in the bracket, the interference term, is proportional to $\cos(\phi_C - \phi_D)$, i.e. it is maximized for the two pumping potentials in phase. Due to the phase-dependent term $\cos\varphi$, the noise correlators show a two-particle Aharonov-Bohm effect, similarly to Ref. [6,7]. It is found that the ac-noise provides no further information about the entanglement.

The orbital entanglement in Eq. (3), focusing on the split quasiparticle pairs detectable via the cross correlators, is independent on whether the electron is emitted towards A and the hole towards B [process (a) in Fig. 2] or vice versa [process (b)]. A Bell Inequality^{3,27} can be formulated in terms of the probability to jointly detect^{5,6} one quasiparticle at A and one at B during a clock-cycle. This probability is formally defined as $P_{ij} = \int_0^\tau dt dt' P_{ij}(t, t')$ with $P_{ij}(t, t') = P_{ij}^{eh}(t, t') + P_{ij}^{he}(t, t') + P_{ij}^{ee}(t, t') + P_{ij}^{hh}(t, t')$ and e.g.

$$P_{ij}^{eh}(t, t') \propto \langle b_i^\dagger(t) b_j^\dagger(t') b_j^h(t') b_i^e(t) \rangle. \quad (9)$$

The quasiparticle operators are defined as $b^e(t) = \int_0^\infty dE \exp(-iEt/\hbar) b(E)$ and $b^h(t) = \int_{-\infty}^0 dE \exp(iEt/\hbar) b^\dagger(E)$. Evaluating the joint detection probability, we find to leading order in δV that $P_{ij} \propto S_{ij}^{dc}$, as anticipated from the discussion below Eq. (7). One can thus formulate a Bell inequality in terms of the period-averaged, zero-frequency noise.^{5,6,22,28} Choosing an optimal⁵ set of scattering angles $\{\theta_A, \theta_B\}$ we arrive at the Bell inequality $2\sqrt{1+\gamma^2} \leq 2$, maximally violated for $\phi_D - \phi_C = \varphi = 0 \bmod 2\pi$. Dephasing as well as nonequal scattering potentials at C and D can be treated in the same way as in Ref. [5].

So far we considered the limit of weak potential oscillations, where only one quanta $\hbar\omega$ is absorbed or emitted by the scattering electrons in regions C and D . Relaxing

this assumption, for arbitrary strong potential modulations, it is no longer possible to write the state emitted by contacts C and D as an excitation of a single electron-hole pair out of the ground state. Instead, the state can be written as a linear superposition of excitations of multiple electron-pairs, describing a complicated multiparticle entanglement. Moreover, while the amplitude of the weak potential state oscillates with the single frequency ω , the strong potential state can have a complicated time-dependence with a sum of amplitudes with oscillation frequencies $n\hbar\omega$.

It is nevertheless possible (here using the Floquet scattering theory of Ref. [14]) to calculate the current and noise. Focusing on the dc-part, the current is zero for the same reasons as in the weak potential case. Considering for simplicity identical scattering potentials at C and D , ($\phi_C = \phi_D$ as well), the noise is given by the same expression as in Eq. (8), with

$$\frac{\hbar S_0}{2e^2} = \int dE \sum_p \left| \sum_{E_n < 0} [t_{0-n}^* r_{p-n} + r_{0-n}^* t'_{p-n}] \right|^2 \quad (10)$$

where $t_{m-n} \equiv t(E_m, E_n)$ etc, with $E_n = E + n\hbar\omega$. Interestingly, the strong potential modulation only modifies the prefactor of the cross-correlations, not the de-

pendence on the angles θ_A, θ_B . This suggests that two-particle entanglement might be postselected^{6,29} by the noise measurement itself. However, a calculation of the joint detection probability P_{ij} shows that it can in the general case, i.e. without considering particular scattering potentials, not be expressed in terms of the noise cross correlator in Eq. (8). This is a consequence of the complicated time-dependence of the emitted state. Moreover, averaging the time-dependent probability $P_{ij}(t, t')$ over a time much longer than the period τ gives a P_{ij} dominated by a quasiparticle current product term, which makes a violation of the Bell Inequality impossible.^{22,28} It is not discussed here whether there are specific strong pumping potentials for which $P_{ij} \propto S_{ij}^{dc}$, allowing a formulation of a Bell Inequality in terms of zero frequency noise.

In conclusion, we have proposed a scheme for generating quasi-particle entanglement by a time-dependent scattering potential. The entanglement is detected by violation of a Bell inequality, formulated in terms of zero-frequency noise. This adiabatic generation of orbitally entangled electron-hole pairs is considered both for weak and strong potential modulations.

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